Analysis of the Parallel Distinguished Point Tradeoff

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The Inversion Problem

$\mathcal{N}$: key space with size N.

$F : \mathcal{N} \to \mathcal{N}$: one-way function

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For a given inversion target $F(x) = y$, Find x.

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Two extreme methods

- Exhaustive search : T=N, M=1,
- Dictionary attack : T=1, M=N,

where T is total online time, M is storage size.

The Inversion Problem

Time Memory Tradeoff(Hellman)

- *Pre-computation phase :*

  pre-compute sufficiently many $(a, F(a))$ pairs, and

  store a digest of the computation in a table **smaller than the complete dictionary.**

- *Online phase :*

  given an inversion target, using the table, find the answer in time **shorter than required by exhaustive search.**

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(ex. $X \equiv 0 \mod t$)

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1. Construct t many DP matrices using F.
   - each chain is set to end on a DP.

$$m \begin{cases} \mathrm{SP}_1 = \circ \xrightarrow{F} \circ \xrightarrow{F} \cdots\cdots \quad \xrightarrow{F} \circ = \mathrm{EP}_1 \ : \ \mathrm{DP} \\ \mathrm{SP}_2 = \circ \xrightarrow{F} \circ \xrightarrow{F} \cdots\cdots \quad \xrightarrow{F} \cdots\cdots \xrightarrow{F} \circ = \mathrm{EP}_2 \ : \ \mathrm{DP} \\ \vdots \\ \mathrm{SP}_m = \circ \xrightarrow{F} \cdots\cdots \xrightarrow{F} \circ \quad = \mathrm{EP}_m : \ \mathrm{DP} \end{cases}$$

: A single DP matrix

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2. Store $\{(SP_j, EP_j)\}_{j=1}^{m}$ only, throwing the rest out.

The DP Tradeoff : online phase

Given an inversion target $y = F(x)$

1. **Online chian creation**

Create *online chain* from y.

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2. **pre-computed chain regeneration**

Expectation :

$$\mathrm{SP}_2 = \circ \xrightarrow{F} \circ \xrightarrow{F} \cdots\cdots \quad {\color{red}x} \xrightarrow{F} y \xrightarrow{F} \circ \cdots\cdots \xrightarrow{F} \overset{\color{red}\text{alarm!!}}{\color{red}\bullet} = \mathrm{EP}_2$$

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pre-computed chain regeneration :

$$\mathrm{SP}_2 = \circ \xrightarrow{F} \circ \xrightarrow{F} \cdots\cdots \xrightarrow{F} x \xrightarrow{F} y$$

'x' is just found!!!

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However,

Most case : Since F is not injective, $\acute{x} \neq x$

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• Whole pre-computed chain is re-generated, but 'x' cannot be found.

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Oechslin

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,where $F_i = r_i \circ F$.

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- The expected number of distinct points in a single DP matrix is $\mathtt{D}_{cr}mt$, where

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$$\mathtt{R}_{tc} = \frac{l^3}{(2l+1)(2l+2)(2l+3)}\bigg(\{(2l-1)+(2l+1)\mathtt{R}_{msc}\}(2+\mathtt{R}_{msc})^2 \\ -4\{(2l-1)+l(2l+3)\mathtt{R}_{msc}\}\left(\tfrac{2}{2+\mathtt{R}_{msc}}\right)^{2l}\bigg).$$

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For each entry (SP_i, EP_i) in the DP tradeoff and the rainbow method,

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- A full record of the online chain is maintained during the online phase,
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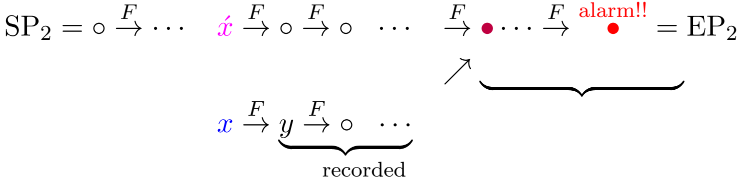

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$$\mathrm{SP}_2 = \circ \xrightarrow{F} \cdots \quad \acute{x} \xrightarrow{F} \circ \xrightarrow{F} \circ \quad \cdots \quad \xrightarrow{F} \bullet \cdots \xrightarrow{F} \overset{\text{alarm!!}}{\bullet} = \mathrm{EP}_2$$

$$x \xrightarrow{F} \underbrace{y \xrightarrow{F} \circ \quad \cdots}_{\text{recorded}}$$

pre-computed chain regeneration :

$$\mathrm{SP}_2 = \circ \xrightarrow{F} \circ \cdots \; \acute{x} \xrightarrow{F} \circ \xrightarrow{F} \circ \cdots \xrightarrow{F} \bullet$$

(Recall : In the original DP tradeoff,

$$\mathrm{SP}_2 = \circ \xrightarrow{F} \cdots \quad \acute{x} \xrightarrow{F} \circ \xrightarrow{F} \circ \cdots \xrightarrow{F} \bullet \cdots \xrightarrow{F} \quad \bullet \quad = \mathrm{EP}_2 \;)$$

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$$\boxed{mt^2 = \mathtt{D}_{msc} N}$$

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- The **online chain creation** of the pD Tradeoff require

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  invocations of F.

- The number of iterations required by the pD tradeoff in **dealing with alarms** is

  $$t^2 \frac{\ln(1-\mathtt{D}_{ps})}{\mathtt{D}_{cr}} \int_0^1 (1-\mathtt{D}_{ps})^{1-u} \ln u \; du.$$

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The *time memory tradeoff curve* for the pD tradeoff is $\mathrm{TM}^2 = \mathtt{pD}_{tc} N^2$,

where

$$\mathtt{pD}_{tc} = \Big(\frac{\ln(1-\mathtt{D}_{ps})}{\mathtt{D}_{ps}} \int_0^1 (1-\mathtt{D}_{ps})^{1-u} \ln u \; du + \frac{1}{\mathtt{D}_{msc}}\Big) \frac{1}{\mathtt{D}_{cr}^3} \mathtt{D}_{ps} \{\ln(1-\mathtt{D}_{ps})\}^2.$$

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- Recall : In the original DP tradeoff,

$$\mathtt{D}_{tc} = (\qquad 2 \qquad + \frac{1}{\mathtt{D}_{msc}}) \frac{1}{\mathtt{D}_{cr}^3} \mathtt{D}_{ps} \{\ln(1-\mathtt{D}_{ps})\}^2.$$

pD versus DP

Since

$$\frac{\ln(1-\mathtt{D}_{ps})}{\mathtt{D}_{ps}} \int_0^1 (1-\mathtt{D}_{ps})^{1-u} \ln u \; du < 1 < 2,$$

$$\text{DP} < \text{pD}$$

the pD tradeoff will outperform the original DP tradeoff.

pD versus Rainbow

- $\mathtt{X}_{tc} = \frac{TM^2}{N^2}$ is a measure of how efficiently the algorithm balances online time against storage requirements.
  - A smaller $\mathtt{X}_{tc}$ implies a more efficient tradeoff.
- However, a better tradeoff efficiency usually requires a higher pre-computation cost and is not always desirable in practice.

$\Rightarrow$ We have to consider both $\mathtt{X}_{tc}$ and $\mathtt{X}_{pc}$ for comparison.

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$\Rightarrow$ We have to consider both $\mathtt{X}_{tc}$ and $\mathtt{X}_{pc}$ for comparison.

- In a fair manner, compare $\mathtt{D}_{tc}$ with $4\mathtt{R}_{tc}$, since $M_R = 2M_D$.

pD versus Rainbow

The pD tradeoff

$$\mathtt{pD}_{tc} = \Big(\frac{\ln(1-\mathtt{D}_{ps})}{\mathtt{D}_{ps}}\int_0^1 (1-\mathtt{D}_{ps})^{1-u}\ln u\ du + \frac{1}{\mathtt{D}_{msc}}\Big)\frac{1}{\mathtt{D}_{cr}^3}\mathtt{D}_{ps}\{\ln(1-\mathtt{D}_{ps})\}^2$$

The rainbow method[HM10]

$$\mathtt{R}_{tc} = \tfrac{l^3}{(2l+1)(2l+2)(2l+3)}\Big(\{(2l-1)+(2l+1)\mathtt{R}_{msc}\}(2+\mathtt{R}_{msc})^2$$
$$-4\{(2l-1)+l(2l+3)\mathtt{R}_{msc}\}\big(\tfrac{2}{2+\mathtt{R}_{msc}}\big)^{2l}\Big)$$

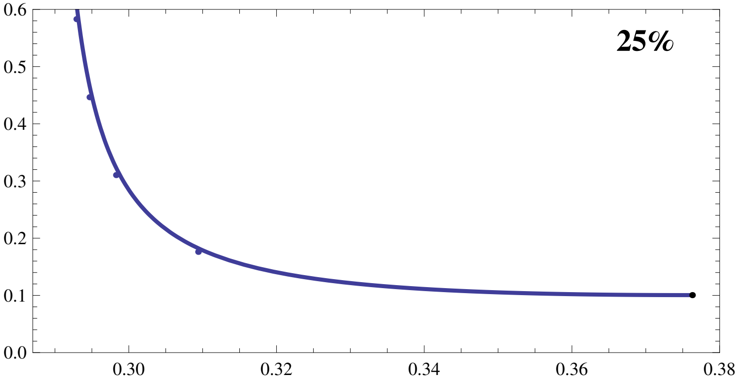
,where

$$\mathtt{R}_{ps} = 1-\Big(\frac{2}{2+\mathtt{R}_{msc}}\Big)^{2l},\ \mathtt{D}_{ps} = 1-e^{-\mathtt{D}_{cr}\mathtt{D}_{pc}}.$$

pD versus Rainbow

$\boxed{\mathtt{D}_{pc} : \mathtt{pD}_{tc} \,,\, \mathtt{R}_{pc} : 4\mathtt{R}_{tc}}$

Figure: the pD(line) and the rainbow(bullet)



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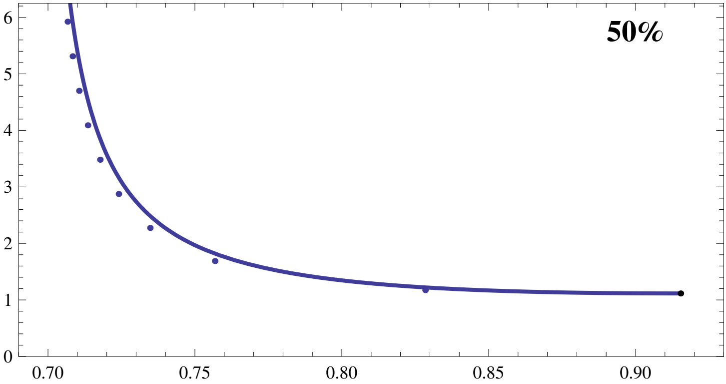

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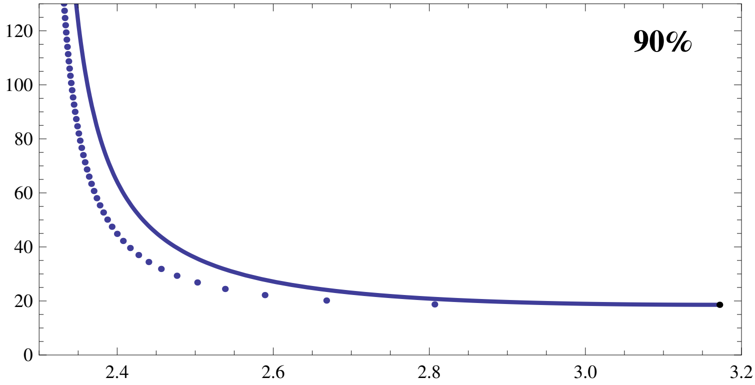

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D_{pc} : pD_{tc} , R_{pc} : $4R_{tc}$

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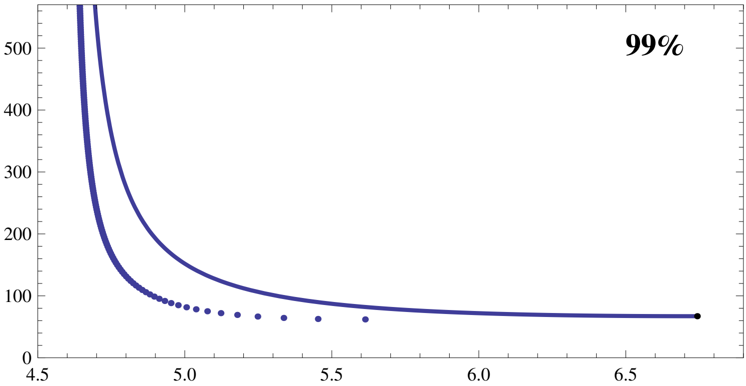

Conclusion

- There are two added conditions in the pD in comparison with the DP.
  - online chain record
  - parallel processing

  $\Rightarrow$ In the online phase, cost for resolving alarms is reduced more than half.
- The pD tradeoff is not likely to be preferable over the rainbow method under most situations.
- The only exception is when the success rate requirement is very low.
  - example. multi-target time memory tradeoff